

## Ignorance Is Bliss: General and Robust Cancellation of Decoherence via No-Knowledge Quantum Feedback

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A “no-knowledge” measurement of an open quantum system yields no information about *any* system observable; it only returns noise input from the environment. Surprisingly, performing such a no-knowledge measurement can be advantageous. We prove that a system undergoing no-knowledge monitoring has reversible noise, which can be canceled by directly feeding back the measurement signal. We show how no-knowledge feedback control can be used to cancel decoherence in an *arbitrary* quantum system coupled to a Markovian reservoir that is being monitored. Since no-knowledge feedback does not depend on the system state or Hamiltonian, such decoherence cancellation is guaranteed to be general and robust, and can operate in conjunction with any other quantum control protocol. As an application, we show that no-knowledge feedback could be used to improve the performance of dissipative quantum computers subjected to local loss.

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“More signal, less noise” is the guiding philosophy of experimental science. Increasing measurement sensitivity is a proven strategy for pushing the frontiers of science and technology, yielding improved knowledge and control over nature. However, at the quantum scale physics pushes *back* by imposing a fundamental limit on the signal-to-noise ratio by virtue of Heisenberg’s uncertainty principle [1,2]. Nevertheless, “more signal, less noise” also guides the design of protocols for the measurement and control of quantum systems, such as squeezed state photon [3] and atom [4] interferometry, optimal parameter estimation [5], weak measurement [6], measurement-based feedback control [5,7], and adaptive measurement [8]. In this Letter, we take the unorthodox “no signal, only noise” approach, and consider measurements that are pure noise, and therefore give no knowledge of the quantum state whatsoever. From a quantum control perspective, one intuitively expects such “no-knowledge” measurements to be unworthy of study, since robust feedback control requires at least some (and preferably good) knowledge of the system state. On the contrary, we show that a measurement-based feedback protocol based on no-knowledge monitoring can be used to remove decoherence—the bane of quantum technology—from an arbitrary quantum system coupled to a Markovian environment that can be monitored.

Although the “no signal, only noise” approach is unorthodox, it has been considered within the context of channel correction. In Refs. [9–11], it was proven that coherence could be recovered in a noisy channel provided the

conditional evolution was random unitary. Consequently, complete correction is, in principle, possible for systems with dimension  $d \leq 3$ . Furthermore, it was proven that measurements that returned a small amount of knowledge (“little signal, mostly noise”) provided a good error correction strategy, and a tradeoff relation between information extraction and correction efficacy was established [12]. Our no-knowledge feedback scheme is consistent with these results; however, it goes several steps further as (1) it concretely shows how decoherence can be canceled in a system of arbitrary dimension, with arbitrary coupling to a Markovian environment, and (2) it provides the explicit physical description of both the measurement and the conditional evolution via our use of the continuous quantum measurement framework.

Attempts to mitigate decoherence have resulted in significant successes, including the development of error correction codes [13–16], dynamical decoupling [17], reservoir engineering [18,19], feedback control [20–24], and the engineering of decoherence-free subspaces [25,26]. Nevertheless, decoherence has yet to be adequately tamed. In our proposal, decoherence is canceled by directly feeding the no-knowledge measurement signal back into the system, in effect turning quantum noise against itself. The scheme only requires knowledge of the decoherence channel to be canceled; no knowledge of the system state is required. It is consequently effective and robust, and can be used in conjunction with other quantum control protocols.

This demonstrates that meaningful feedback control without knowledge is not only possible, but desirable.

*No-knowledge measurements.*—Consider a system with Hamiltonian  $H$  that interacts with a Markovian reservoir via the coupling operator  $L$ . The system density operator  $\rho_t$  evolves according to the master equation (ME)

$$\partial_t \rho_t = -i[H, \rho_t] + \mathcal{D}[L]\rho_t \equiv \mathcal{L}\rho_t, \quad (1)$$

where  $\partial_t \equiv d/dt$ ,  $\mathcal{D}[Z]\rho_t = Z\rho_t Z^\dagger - (Z^\dagger Z\rho_t + \rho_t Z^\dagger Z)/2$ , and we have set  $\hbar = 1$ . In principle, it is always possible to indirectly extract information about the system with a projective measurement on the reservoir. In particular, for a homodyne measurement of the environment at angle  $\theta$ , the conditional system dynamics are described by the Stratonovich stochastic master equation [5,27,28]

$$\partial_t \rho_t = \mathcal{L}\rho_t + \sqrt{\eta} \mathcal{A}[Le^{i\theta}] \rho_t y_\theta(t) - \frac{\eta}{2} \mathcal{A}^2[Le^{i\theta}] \rho_t, \quad (2)$$

where  $\rho_t$  is the unnormalized conditional density operator for the system,  $\eta$  is the detection efficiency,  $\mathcal{A}[Z]\rho_t = Z\rho_t + \rho_t Z^\dagger$ , and  $\mathcal{A}^2[Z]\rho_t = Z(\mathcal{A}[Z]\rho_t) + (\mathcal{A}[Z]\rho_t)Z^\dagger$ . Conditional expectations of system operators are calculated using  $\langle X \rangle_t = \text{Tr}[X\rho_t]/\text{Tr}[\rho_t]$ . The first term of Eq. (2) corresponds to the unconditional Lindblad ME (1) and gives the unitary dynamics due to the system Hamiltonian and the decoherence caused by the system-reservoir coupling. The second term is the *innovations*, which conditions the system dynamics on the homodyne measurement photocurrent

$$y_\theta(t) = \sqrt{\eta} \langle Le^{i\theta} + L^\dagger e^{-i\theta} \rangle_t + \xi(t), \quad (3)$$

where  $\xi(t)$  is a Stratonovich stochastic integral [29,30]. The final term of Eq. (2) is the Stratonovich correction [31]. Equation (1) is obtained by averaging Eq. (2) over different realizations of the measurement record, up to a normalization factor.

Equation (3) shows that the measurement signal is composed of two parts: the first term represents the knowledge obtained about the system from the measurement, whereas the second term is the corrupting quantum (white) noise input from the reservoir. However, there exist choices of  $L$  for which the measurement returns no information about the system operators, which we term a no-knowledge measurement. Specifically, when  $L$  is Hermitian, homodyne detection of the reservoir at angle  $\theta = \pi/2$  is a no-knowledge measurement, since the measurement signal  $y_{\pi/2}(t) = \xi(t)$  returns only noise. No-knowledge monitoring appears in early works on continuous quantum measurement as a means of obtaining simpler linear stochastic MEs [32,33], in the investigation of the localization properties of conditioned states [34], and in the discussion of state estimation [35,36].

We can examine the effect of a no-knowledge measurement by comparing the evolution of the underlying system state  $\rho_t$  to that of the quantum filter  $\pi_t$ , which is the optimal Bayesian estimate of the system state conditioned on the measurement record [31]. The unnormalized quantum filter  $\pi_t$  evolves according to [38,39]

$$\partial_t \pi_t = \mathcal{L}\pi_t + \sqrt{\eta} \mathcal{A}[Le^{i\theta}] \pi_t y_\theta(t) - \frac{\eta}{2} \mathcal{A}^2[Le^{i\theta}] \pi_t. \quad (4)$$

Suppose that we have the situation shown in Fig. 1(a) (without the feedback) where the system is prepared in the state  $\rho_0$  and evolves according to Eq. (2), while an observer, ignorant of the underlying system state, models the system

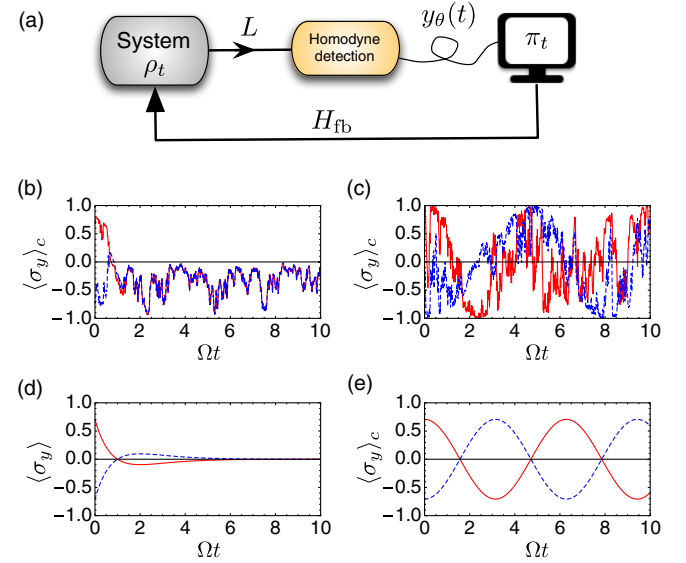


FIG. 1 (color online). (a) Schematic for a measurement-based feedback control protocol. In general, information about the system  $\rho_t$  is extracted by monitoring the decoherence channel  $L$ . The optimal estimate  $\pi_t$  is conditioned on the resulting measurement signal  $y_\theta(t)$ . The system is then controlled with some feedback Hamiltonian  $H_{fb}$ . For our no-knowledge feedback protocol, the feedback is simply a modulation of the no-knowledge measurement signal. (b)–(e) Particular example of a driven qubit undergoing dephasing with  $H = \Omega\sigma_x$ ,  $L = \sqrt{\gamma}\sigma_z$ , and  $\Omega/\gamma = 1$ . (b),(c) Conditional trajectories for  $\langle\sigma_y\rangle_c$  when the channel is being monitored (no feedback) with perfect homodyne detection at angles  $\theta = 4\pi/5$  and  $\pi/2$ , respectively. Solid red lines represent the dynamics starting from the underlying initial state  $\rho_0 = [I + (\sigma_x + \sigma_y)/\sqrt{2}]/2$ , while dashed blue lines represent the filter evolution from the (incorrect) initial estimate  $\pi_0 = [I + (\sigma_x - \sigma_y)/\sqrt{2}]/2$ . Although the estimate  $\pi_t$  converges to  $\rho_t$  in (b), in the no-knowledge case (c)  $\rho_t$  and  $\pi_t$  never converge. (d) Dephasing effect for the unmonitored system [cf. Eq. (1)]. (e) Dephasing is canceled by directly feeding back the no-knowledge measurement via the Hamiltonian  $H = \Omega\sigma_x + \sqrt{\gamma}\sigma_z y_{\pi/2}(t)$ . Despite the filter's inaccurate estimate of  $\rho_t$ , decoherence is completely removed, demonstrating that accurate knowledge of the system is not required for effective decoherence cancellation.

by Eq. (4) with  $\pi_0 \neq \rho_0$ . In general, information about the system is extracted from the measurement signal and used to update the observer's estimate. This leads to a better estimate of the system state over time, and  $\pi_t$  converges to  $\rho_t$  in finite time [Fig. 1(b)]. This is *not* true for a no-knowledge measurement, since the filter is conditioned only on noise. Then Eqs. (2) and (4) decouple, and the filter never converges to the system state [31] [Fig. 1(c)].

*Canceling reservoir noise with no knowledge.*—In classical control theory, a system-observation pair is called unobservable if the initial system state cannot be determined from the measurement signal. A system undergoing a no-knowledge measurement is clearly unobservable, as neither the past nor present system state can be determined from the measurement record. One may expect, therefore, that this lack of knowledge renders meaningful measurement-based feedback control impossible. This intuition is incorrect. Although a no-knowledge measurement produces a signal with no dependence on any system observable, the quantum noise that constitutes the signal is precisely the same noise that corrupts the system state. Consequently, by applying an appropriate feedback the no-knowledge measurement signal can be used to cancel the noise corrupting the system's evolution.

Specifically, suppose  $L$  is Hermitian, and we make a measurement of the no-knowledge quadrature  $\theta = \pi/2$  with perfect efficiency  $\eta = 1$ . Then Eq. (2) takes the simple form:

$$\partial_t \rho_t = -i[H - Ly_{\pi/2}(t), \rho_t]. \quad (5)$$

Since the dynamics due to the reservoir noise are unitary, their effect is *reversible* and can be entirely canceled by directly feeding back the measurement signal. Explicitly, by making the replacement  $H \rightarrow H + Ly_{\pi/2}(t)$ , Eq. (5) reduces to  $\partial_t \rho_t = -i[H, \rho_t]$ .

What is particularly interesting about no-knowledge feedback is that it works when the system and filter are initially very different [see Figs. 1(d) and 1(e)]. The reason is that the measurement signal is simply fed back via the Hamiltonian without *any* prior filtering. Indeed, no-knowledge feedback can be successfully implemented with almost no *a priori* knowledge of the underlying system state or dynamics. No-knowledge feedback only requires a correct identification of the no-knowledge quadrature, which depends only on the coupling operator  $L$ , and the ability to monitor this decoherence channel. *A precise description of the system state and its unitary evolution is not required.* This natural robustness [40] gives no-knowledge feedback an advantage over other state-dependent methods of decoherence reduction [41], particularly for systems where the dynamics cannot be precisely quantified.

When the detection efficiency is imperfect, the effectiveness of no-knowledge feedback is reduced. The evolution is no longer purely unitary,

$$\partial_t \rho_t = -i[H - \sqrt{\eta}Ly_{\pi/2}(t), \rho_t] + (1 - \eta)\mathcal{D}[L]\rho_t, \quad (6)$$

and therefore cannot be entirely canceled by feeding back the measurement signal. Nevertheless, by choosing the no-knowledge feedback  $H \rightarrow H + \sqrt{\eta}Ly_{\pi/2}(t)$ , the decoherence rate can be reduced by a factor of  $(1 - \eta)$  [cf. Eq. (1)]:

$$\partial_t \rho_t = -i[H, \rho_t] + (1 - \eta)\mathcal{D}[L]\rho_t. \quad (7)$$

Experiments with imperfect detection efficiency can therefore still enjoy a significant and robust decoherence reduction by employing no-knowledge feedback.

An analogous result exists for photodetection, where unitary  $L$  corresponds to a no-knowledge measurement. Noise is canceled by applying a unitary gate to the system after the detection of a photon [31].

*Removing decoherence for general  $L$ .*—As formulated above, a no-knowledge measurement is only possible when the coupling operator is Hermitian [42]. Since physical observables are Hermitian, direct no-knowledge measurements are possible in many situations. Examples include dephasing in qubits ( $L = \sigma_z$ ) [43], optomechanical devices under position measurement ( $L = x$ ) [44], and minimally destructive detection of Bose-Einstein condensates [22–24,45]. However, some common coupling operators, such as the annihilation operator  $a$ , are not Hermitian. Fortunately, we can still remove decoherence for a general  $L$  via a similar measurement-based feedback scheme. Counterintuitively, this requires an extra reservoir with coupling operator  $L^\dagger$ , giving the unconditional dynamics

$$\partial_t \rho_t = -i[H, \rho_t] + \mathcal{D}[L]\rho_t + \mathcal{D}[L^\dagger]\rho_t. \quad (8)$$

The “trick” is to recognize that  $\mathcal{D}[L]\rho_t + \mathcal{D}[L^\dagger]\rho_t = \mathcal{D}[L_+]\rho_t + \mathcal{D}[L_-]\rho_t$ , where  $L_\pm = i^{(1\mp 1)/2}(L \pm L^\dagger)/\sqrt{2}$  are Hermitian. Thus,  $L_\pm$  are effective coupling operators that admit no-knowledge measurements.

Measurements of  $L_\pm$  are possible by taking the output channels of both reservoirs, mixing them via a 50:50 beam splitter, introducing a relative phase shift of  $\pi/2$ , and subsequently measuring each output with homodyne detection (see Fig. 2). This yields the two measurement signals  $y_\theta^\pm(t) = 2\sqrt{\eta}\cos\theta\langle L_\pm \rangle_t + \xi_\pm(t)$ , where  $\xi_\pm(t)$  are independent Stratonovich noises. No-knowledge measurements of  $L_\pm$  occur for quadrature angle  $\theta = \pi/2$ . The beam splitting step of the feedback protocol is vital, and has no classical analogue, making our result a *quantum* feedback protocol.

The evolution of  $\rho_t$  under these no-knowledge measurements is given by a straightforward generalization of Eq. (6):

$$\begin{aligned} \partial_t \rho_t = & -i[H - \sqrt{\eta}(L_+y_{\pi/2}^+(t) + L_-y_{\pi/2}^-(t)), \rho_t] \\ & + (1 - \eta)(\mathcal{D}[L]\rho_t + \mathcal{D}[L^\dagger]\rho_t). \end{aligned} \quad (9)$$

Finally, we directly feed the measurement signals back via  $H \rightarrow H + \sqrt{\eta}(L_+y_{\pi/2}^+(t) + L_-y_{\pi/2}^-(t))$ :

$$\partial_t \rho_t = -i[H, \rho_t] + (1 - \eta)(\mathcal{D}[L]\rho_t + \mathcal{D}[L^\dagger]\rho_t). \quad (10)$$



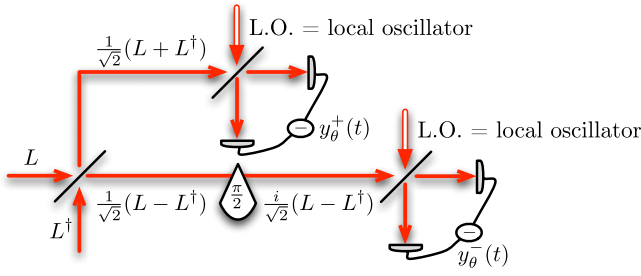


FIG. 2 (color online). Scheme for engineering measurements of  $L_{\pm}$  from the outputs of couplings  $L$  and  $L^{\dagger}$ .

The original decoherence in the system has been suppressed by the factor  $(1 - \eta)$ , admittedly at the cost of introducing additional decoherence due to  $L^{\dagger}$ . However, in the perfect detection efficiency limit,  $\eta \rightarrow 1$ , all decoherence is eradicated from the system.

The successful implementation of our scheme requires some level of reservoir engineering and monitoring. In principle, such dissipative engineering is possible for a range of physical systems. For example, Carvalho and Santos [46] showed how to engineer an additional  $\sigma_{+}$  reservoir to the spontaneous emission decoherence of two qubits. This system is a specific instance of Eq. (9) for  $L = \sigma_{-}$ , and allows for the protection of entanglement via the environmental monitoring [46–49], or even quantum computation when applied to multiple qubits [50]. Although none of the papers considered the possibility of canceling decoherence via no-knowledge feedback, implementing such feedback would be straightforward via the inclusion of the feedback Hamiltonian  $H = (\sigma_{-} + \sigma_{+})y_{\pi/2}^{+}(t)/\sqrt{2} + i(\sigma_{-} - \sigma_{+})y_{\pi/2}^{-}(t)/\sqrt{2}$ . This Hamiltonian simply corresponds to the application of two classical fields resonant to the qubit transition and modulated by the measurement signals.

From an experimental standpoint, the homodyne monitoring and modulated feedback driving should be relatively simple to implement in a variety of physical systems. In particular, specific homodyne quadratures can be chosen with a high degree of accuracy, as is routinely done in tomography, and with high efficiencies. More challenging is the reservoir engineering step and the efficient collection of the decoherence channel, which ultimately limits the overall efficiency  $\eta$ . Nevertheless, recent demonstrations in systems as diverse as superconducting qubits [51–53], cavity QED experiments [54], and ion traps [55,56] indicate that an experimental realization of our scheme is entirely plausible in the near future. For example, Ref. [52] reported  $\eta = 0.49$  when monitoring a cavity field coupled to a superconducting qubit, and efficiencies above 90% are achievable via coupling an ancilla to the superconducting qubits [57]. In microwave cavity experiments, cavity field monitoring with  $\eta = 0.5$  has been demonstrated [58].

*Application: Dissipative quantum computing.*—It was recently shown that appropriately engineered quasilo-

dissipation can be used to perform universal quantum computation (UQC) [59,60]. Although such dissipative quantum computing (DQC) is robust to decoherence in principle, in practice it is likely to suffer from local errors due to the presence of local loss. For traditional UQC, local errors can be corrected via quantum error correction (QEC) codes. Indeed, the threshold theorem proves that traditional UQC can be scaled to large numbers of qubits, even when local errors are present, provided QEC is in operation [43]. However, QEC requires precisely timed projective measurement and conditional operations; hence, adding this capacity to DQC greatly complicates the engineering of these systems [61].

We provide a simpler solution. Provided the cause of the local errors is diagnosable, no-knowledge feedback can be used to remove their effect. Crucially, the feedback will work concurrently with any quantum computation. To show this, we consider the effect of local loss on a DQC algorithm designed to generate a linear cluster state [see Fig. 3(a)]. A series of  $N$  qubits evolve under the influence of quasiloal dissipators  $Q_i = \sqrt{\alpha}(1 + \sigma_z^{i-1}\sigma_x^i\sigma_z^{i+1})\sigma_z^i/2$  [with special cases  $Q_1 = \sqrt{\alpha}(1 + \sigma_x^1\sigma_z^2)\sigma_z^1/2$  and  $Q_N = \sqrt{\alpha}(1 + \sigma_z^{N-1}\sigma_x^N)\sigma_z^N/2$  at the boundaries] and local loss operators  $L_i = \sqrt{\gamma}\sigma_{-}^i$ , such that the ME for the whole system is  $\partial_t \rho_t = \sum_i^N (\mathcal{D}[Q_i] + \mathcal{D}[L_i])\rho_t$ . The steady state  $\rho_{ss}$  for the system when there is no local loss ( $\gamma = 0$ ) is a cluster state,  $\rho_{ss} = \rho_{\text{cluster}}$ . However, when local loss is present ( $\gamma \neq 0$ ), the steady state of the system is no longer the target cluster state. As shown in Fig. 3(c), the fidelity  $\mathcal{F} = \sqrt{\text{Tr}[\rho_{ss}\rho_{\text{cluster}}]}$  between the target cluster state and the actual steady state rapidly decreases with system size. However, when no-knowledge feedback is implemented as depicted in Fig. 3(b), the decline in the fidelity as a function of system size is arrested. Engineering the additional local dissipator  $\sigma_{+}^i$  [46,47] required for this feedback should be trivial in comparison to engineering the quasiloal dissipators  $Q_i$ . Figure 3(c) quantifies the effectiveness of the no-knowledge feedback, demonstrating that the fidelity improves as the detection efficiency increases, with the creation of a perfect cluster state possible when  $\eta = 1$ . In fact, since no-knowledge feedback can operate concurrently with any DQC algorithm, it could be included in addition to QEC. Hence, no-knowledge feedback with an imperfect detection efficiency may reduce the error rate to the threshold required for truly scalable DQC.

DQC is just one of many possible quantum technologies that could be improved, or made possible, by the general and robust reduction of decoherence via no-knowledge feedback. However, since no-knowledge feedback can operate in conjunction with other quantum control protocols, it does not compete with other decoherence reduction methods (e.g., QEC), but rather complements them. Furthermore, given the simplicity of no-knowledge feedback, we suspect that no-knowledge *coherent* feedback control is a strong

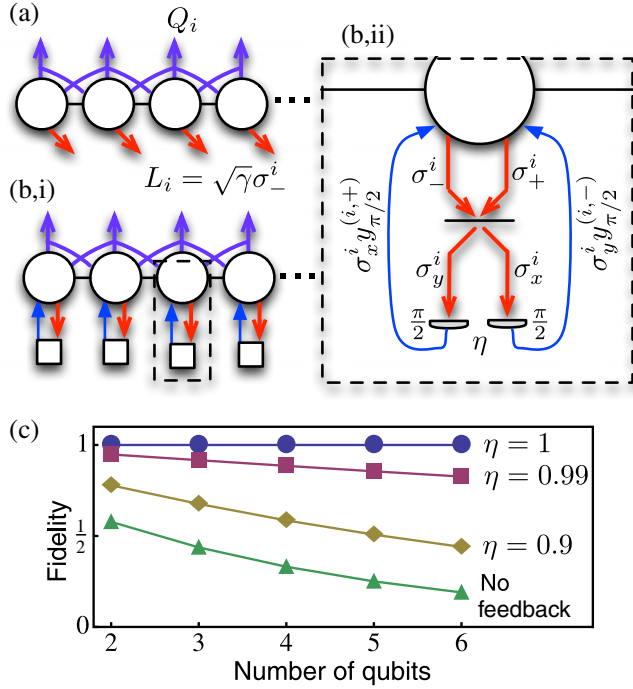


FIG. 3 (color online). (a) A DQC setup with an  $N$  qubit chain coupled to quasilocals  $Q_i$  and local loss operators  $L_i$ . As demonstrated in (c), a loss rate of  $\gamma/\alpha = 10$  decreases the fidelity between the target cluster state and the system steady state (green triangles), and decreases it more severely for a larger number of qubits. (b, i) The errors introduced by the local loss are corrected by applying our no-knowledge feedback protocol on each qubit. (b, ii) For each qubit a no-knowledge measurement is constructed by coupling an additional reservoir  $\sqrt{\gamma} \sigma_+^i$  and measuring  $\sigma_x^i$  and  $\sigma_y^i$  at a homodyne angle of  $\pi/2$  as summarized in Fig. 2. Decoherence is canceled by feeding back  $H = \sqrt{\eta\gamma} \sum_i [\sigma_x^i y_{\pi/2}^{(i,+)}(t) + \sigma_y^i y_{\pi/2}^{(i,-)}(t)]$ . (c) The fidelity as a function of system size for no feedback (green triangles), and no-knowledge feedback with detection efficiency  $\eta = 0.9$  (yellow diamonds),  $\eta = 0.99$  (red squares), and  $\eta = 1$  (blue circles).

possibility. The many advantages of no-knowledge feedback strengthen the case for more reliable and robust dissipation engineering, as this is a vital ingredient for the cancellation of general forms of decoherence.

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